Why is there a gap in social networks efficiency between minority and majority groups in the labor market?

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We develop a labor matching model with social networks where we study the correlation between networks efficiency and workers' origins - or types. Contacts are endogenously created and made up of individuals of two origins. Simultaneous variations of three key elements, the relative number of individuals of each type in the population, average preferences for same-type ties and biases in meetings, affect the average composition of social networks for each types. Considering information transfer on vacant jobs, since there are a majority and a minority group, we describe the mechanism by which variations in the three key elements we above-mentioned affect labor market outcomes. Our main results show that a rise in preferences for same-type ties always increases inequalities between the two groups whereas a rise in meeting biases, which could be compared to a rise in segregation, has a more ambiguous effect. Indeed, a rise in meeting biases could have a positive impact on labor market outcomes for members of the two groups. This positive effect could change for members of the minority group since they are subjected to some hiring penalty.

Key Words: Social Networks, Hiring Penalty, Homophily, Segregation, Unemployment.

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1. INTRODUCTION

According to many empirical studies on OECD countries, migrants and their descendants have on average higher unemployment rates and their access to employment varies with their origin. Researches have for a long time focused on the hiring discrimination phenomenon in order to explain why workers with the same level of education but different origin have on average different unemployment rates. Yet, a growing set of research shows that communication through social networks (SN) about vacant jobs plays an important role in the formation of inequalities in the labor market according to the origin of individuals. Domingues Dos Santos (2005) has for instance highlighted that in France a part of inequalities in the labor market between the North African and the Portuguese community comes from differences in SN. Holzer (1987) as well as Frijters & al (2005) have respectively shown that in the U.S. and in the U.K. a part of ethnic inequalities in the labor market is related to heterogeneity in the efficiency of SN. Where does this difference in the efficiency of SN come from?

A few empirical studies have pointed out that some factors, which could influence the way SN form, play an important role in the explanation of ethnic inequality in the labor market. In fact, we think that variations in three factors, that besides Currarini, Jackson and Pin (2009) (CJ&P) have shown determine the way SN form, influence labor market outcomes for minorities. First, Battu & al (2007) pointed out that the average attachement to the culture of origin could play an important role in explaining labor market outcome for the minority in the U.K.. They measure the attachment to the culture of origin through questions about Britishness of individuals from ethnic minorities. As for the contacts formation side, CJ&P have shown that variation in preferences for same-type ties, which is in some way equivalent to variation in the attachment to the culture of origin, influences the way SN form. Second, since there are preferences for same-type ties, the relative number of individuals of each type in the population matters. Munshi (2003), without really explaining the mechanisms in actions, has among other things shown that in the U.S. labor market, the larger is the size of the mexican community in a labor pool, the more efficient are SN for members of this community. From the network formation side, CJ&P also insist on the role of the relative number of individuals of each type in the population. Finally, Conley and Topa (2002) as well as Edin & al (2003) and Patacchini and Zenou (2008) have shown that the average geographical distance between members of a giving ethnic community plays an important role in explaining their situation in the labor market, more than their physical distance from jobs or than their level of education. This could be compared to type-biases in meeting opportunities, the third factor that CJ&P introduce to explain SN formation in a population with individuals of various origins.

In fact, none of the empirical studies above-mentioned about factors which play a role in the explanation of ethnic inequality in the labor market gives a clear explanation of the way these factors, through SN, affect labor market outcomes. Moreover, these empirical studies deal with the three factors we above-refered to separatly whereas, as CJ&P shows, the relative size of each group, preferences for same-type ties and meeting biases may influence the way SN form in contradctory ways. Introducing CJ&P's model of network formation between individual of various origin in a labor market model could then help us better understand how determinants of SN affect labor market outcomes for minorities. Moreover, using CJ&P's modelisation is all the more relevent as they rely upon data on connexions between individuals of various origins which is relatively uncommon. Indeed, it is difficult to rigorously endogenize network formation. Even if their data concern contacts between U.S. high school students, many studies confirm that individuals tend on average to associate with individuals of the same origin (Grosseti, 2007, Lewis & al, 2008), this phenomenon being increasing with age (Marsden, 1988).

We then rely upon both the model of Ioannides and Soetevent (2006) (I&S) and the model of Fontaine (2008) in order to introduce SN in a matching model *à la* Pissarides (2000). Individuals get ties in the first period, then, as in I&S, they enter the labor market. Moreover, we choose, as I&S, a discret time model. Finally, we use the same matching function as Fontaine (2008) to make the simulation more simple.

In the first part of our model we briefly illustrate, relying upon CJ&P's model, how SN are shaped, especially how simultaneous variations in the relative size of both groups, in preferences for same-type ties and in meeting opportunity biases influence both the average homophily rate for each type (the percentage of same-type ties an individual has) and each type average total number of contact per individual. In a second part we study how, when information about vancant jobs are partly distributed through contacts, a rise in the average minority group individuals' preferences for same-type ties, which could be compared to a lack of cultural integration to the host country, bear upon labor market outcomes for members of this group. Conversely, since preferences for same-type ties exist, a rise in meeting biases, which could be compare to a rise in segregation, improves labor market outcomes for member of the minority group. Finally, we show in the third part, as Arrow (1998) suggested, that the impact of SN on minority's situation in the labor market should not be studied separatly from the study of hiring discrimination or any hiring penalty. Indeed, if considering hiring penalty for members of the minority group does not roughly change the way preferences for same-type ties influence labor market outcomes for these last, a rise in segregation could now have a negative effect on the situation of the minority members in the labor market. We then conclude that our theoretical framework allow us to make better political recommendations than the empirical studies we above-mentioned.

2. NETWORK FORMATION

We assume that there are two types of individuals i and j, (with two differents origins). We then have a population $N = N_i + N_j$ where $n_i = \frac{N_i}{N}$ and $n_j = \frac{N_j}{N}$ (N_i is the number of individual of type i, N_j is the number of individual of type j and N the total number of individuals in the population). Individuals of type i form ties in function of the relative size of there group n_i in the population, but also in function of their preference to form tie with their type i and in function of their meeting opportunity biases (going to cultural institutions, meeting through friends, etc.).

2.1. The basic mecanism

We consider a matching process between individuals *à la* CJ&P. Each individual maximize his utility $U(s_i, d_i)$ which is increasing and concave and where s_i and d_i are respectively for an individual of type i the number of contact of the same type i and of the other type j. In fact, N_i individuals of type i enter in the matching process and form one tie in each period, the same for the N_j individuals of type j. According to his utility and his constraint, each individuals decides to enter l times in the process to get contacts. At each time individuals meet other individuals of both types. In the end of the process individuals of type i went on average l_i times in the process and have each formed l_i ties with $l_i = s_i + d_i$. Then, all individual of type i considered have formed $N_i \times l_i$ contacts.

2.2. Preferences and meeting opportunity biases

2.2.1. Preferences for the same type

We consider the following utility function, $U(s_i, d_i) = (s_i + \gamma_i d_i)^{\alpha_i}$ where $\gamma_i \in [0, 1]$ depreciates the utility of a contact with an individual of type j for individuals of type i and $\alpha_i \in [0, 1]$ is a coefficient which catches the decreasing marginal utility of the total number of contacts. We will consider $\alpha_i = \alpha_j = \alpha$. Doing so, we consider that type i and type j will have the same satisfaction for the total number of ties they form. Then, when $\gamma_i < 1$, each type give more value to a contact with an individual of the same type.

Individuals of type i choose to enter in the matching process l_i times in function of both $U(s_i, d_i)$ and the cost c_i of entering the process. They meet an individual of the same type with probability q_i each time they enter the process and an individual of the other type with probability $(1 - q_i)$.

Knowing q_i, probability to meet someone of the same type, we have $s_i = l_i q_i$ and $d_i = l_i(1 - q_i)$. Then

$$U(l_i q_i, l_i(1-q_i)) = (l_i q_i + \gamma_i l_i(1-q_i))^\alpha \tag{1}$$

If $c_i \in [0,1]$ is the cost to enter the process, individuals solve the following program:

$$\underset{l_i}{Max}\left[U(l_i q_i, l_i(1-q_i)) - c_i l_i\right] \tag{2}$$

As for α we assume that $c_i = c_j = c$.

From (2) and (1) we find that the optimal number of contact l_i^* for an agent of type i is

$$l_i^* = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \left(\left(1-\gamma_i\right) q_i + \gamma_i\right)^{\frac{\alpha}{1-\alpha}} \tag{3}$$

Note that $\frac{\partial l_i^*}{\partial \gamma_i} = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \underbrace{(1-q_i)}_{\geq 0} \left(\underbrace{(1-\gamma_i)}_{\geq 0} q_i + \gamma_i\right)^{\frac{2\alpha-1}{1-\alpha}} \iff \frac{\partial l_i^*}{\partial \gamma_i} \geq 0$, that is to say the number of contact rise when preferences decrease ($\gamma_i \to 1$). In the same way, the number of contact rises when $q_i \to 1$ because as we see $\frac{\partial l_i^*}{\partial q_i} = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \underbrace{(1-\gamma_i)}_{\geq 0} \left(\underbrace{(1-\gamma_i)}_{\geq 0} q_i + \gamma_i\right)^{\frac{2\alpha-1}{1-\alpha}} \iff$ $\frac{\partial l_i^*}{\partial q_i} \geq 0$.

2.2.2. Biases in meeting opportunity

Without meeting bias, since there is a large number of agents of each type, individuals of type i and those of type j have the same probability to meet an individual of type i. $q_{ii} = q_{ji} = q_i = \frac{M_i}{M}$ represent this probability, where $M_i = N_i l_i^*$ is the total number of matching individuals of type i make and M is the total number of matching individuals of both type i and j make soo that $M = N_i l_i^* + N_j l_j^*$. In the same way we should also have $q_{jj} = q_{ij} = q_j = \frac{M_j}{M}$.

As CJ&P we introduce biases in meeting opportunity for each type toward individuals of the same type. This meeting bias could for instance represent the more or less important average physical or geographical distance between individual of the same type as mentionned in Conley and Topa (2002). If q_i and q_j are respectively the probability for types i and j to meet individuals of the same type, then, instead of having $q_i + q_j = 1$, which would be logical with $q_i = \frac{M_i}{M}$ and $q_j = \frac{M_j}{M}$ because $M_i + M_j = M$, we have

$$q_i^{\beta_i} + q_j^{\beta_j} = 1 \tag{4}$$

where $\beta_i > 0$ represent meeting biases for type i and $\beta_j > 0$ meeting biases for type j. Probabilities to meet individual of the other type are respectively $(1 - q_i)$ for individuals of type i and $(1 - q_j)$ for individuals of type j.

$\beta_i > 1$ implies for instance for individuals of type i that they will have higher probability to meet individual of the same type than at random. When $\beta_i > \beta_j$, biases in meeting opportunity are higher for type i.

2.3. Additionnal condition

Since the total number of contacts formed by individuals of type i with individuals of type j is by definition the same as the total number of contacts formed by individuals of type j with type i, we have

$$(M_i)(1 - q_i) = (M_j)(1 - q_j)$$

$\Longleftrightarrow$

$$(N_i l_i^*)(1 - q_i) = \left(N_j l_j^*\right)(1 - q_j)$$

$\Longleftrightarrow$

$$n_i l_i^* (1 - q_i) = (1 - n_i) l_j^* \left(1 - \left(1 - q_i^{\beta_i}\right)^{\frac{1}{\beta_j}}\right) \tag{5}$$

2.4. Homophily rate and density of networks for each type

Individuals of type i finally form on average $s_i + d_i$ contacts. In the same way individuals of type j form $s_j + d_j$ contacts. From all the previous equations we have

$$\begin{aligned} s_i &= s_i(N_i, \gamma_i, \beta_i) = l_i^* \times q_i \\ d_i &= d_i(N_i, \gamma_i, \beta_i) = l_i^* \times (1 - q_i) \end{aligned} \tag{6}$$

and

$$\begin{aligned} s_j &= s_j(N_j, \gamma_j, \beta_j) = l_j^* \times q_j \\ d_j &= d_j(N_j, \gamma_j, \beta_j) = l_j^* \times (1 - q_j) \end{aligned} \tag{7}$$

We fix $\gamma_i, \beta_i, \gamma_j$ and β_j. We have to find $l_i^* (n_i)$, $q_i (n_i)$, $l_j^* (n_i)$ and $q_j (n_i)$. To do so, we rest on (1), (3), (4) and (5) and solve the following system:

$$\begin{cases} l_i^* = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \left((1-\gamma_i) q_i + \gamma_i\right)^{\frac{\alpha}{1-\alpha}} \\ q_i^{\beta_i} + q_j^{\beta_j} = 1 \\ l_j^* = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} \left(\left(1-\gamma_j\right)\left(1-q_i^{\beta_i}\right)^{\frac{1}{\beta_j}} + \gamma_j\right)^{\frac{\alpha}{1-\alpha}} \\ n_i l_i^* (1-q_i) = (1-n_i) l_j^* \left(1-\left(1-q_i^{\beta_i}\right)^{\frac{1}{\beta_j}}\right) \end{cases} \tag{8}$$

We cannot analytically solve (8) to obtain l_i^*, q_i, l_j^* and q_j, that is why as CJ&P we use simulation. We fix α and c such that the total number of contacts each individual finally forms is included between 20 and 40. This correspond to current figures of the literature (see Fontaine, 2008, Ioannides et Soetevent, 2006).

We represent in the Fig.1 homophily curves $H_i (n_i) = \frac{s_i}{s_i + d_i}$ and in the Fig.2 the representative curves $l_i (n_i)$ of the number of contacts formed by individuals of type i, with γ and β the same for the two types. In the Fig.1 we observe the impact of simultaneous variation in n_i and γ, and also in n_i and β on $H_i (n_i)$. We see that H_i is always increasing in n_i. Moreover, when $\beta > 1$, a rise in preferences ($\gamma \to 0$) have a negative impact on H_i if type i is the minority. But when $0 \leq \gamma < 1$, a rise in β has a positive impact on H_i. When $0 \leq \gamma < 1$ and $\beta > 1$, $H_i (n_i)$ is an increasing and concave function.

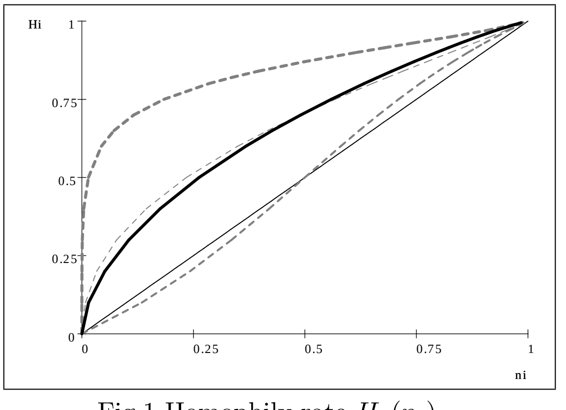

Fig.1 Homophily rate $H_i (n_i)$.

No biases, γ=1 and β=1 (black thin solid).
Weak biases, γ=0.5 and β=2 (gray thin dashes).
Biases in preferences only, γ=0.5 and β=1 (gray medium dashes).
Strong meeting biases, γ=0.5 and β=5 (gray thick dashes).
Stong biases, γ=0.1 and β=2 (black thick solid).

The total number of contacts an individual has varies more or less in the same way as $H_i(n_i)$. l_i is always increasing in n_i. Moreover, when $\beta > 1$, a rise in preferences ($\gamma \to 0$) have a negative impact on l_i but unlike for $H_i(n_i)$, not only if type i is the minority. Finally, when γ exists, a rise in β has a positive impact on l_i. We furthermore note something different from H_i, it is worst for l_i to be subjected to a rise in preferences when $\beta > 1$ (see the thick black solid curve) than when $\beta = 1$ (see the medium gray dashed curve).

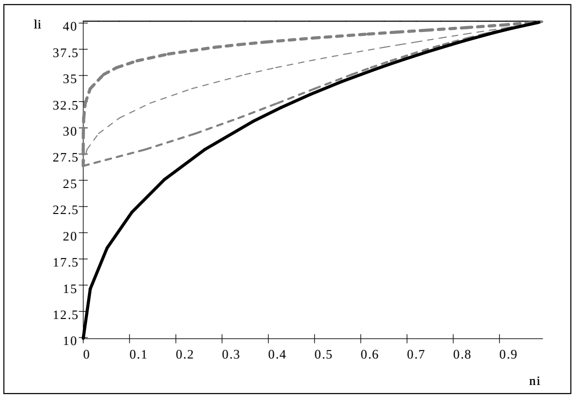

Fig.2 Average number of contact $l_i(n_i)$ for individuals of type i.

No biases, γ=1 and β=1 (black thin solid).
Weak biases, γ=0.5 and β=2 (gray thin dashes).
Biases in preferences only, γ=0.5 and β=1 (gray medium dashes).
Strong meeting biases, γ=0.5 and β=5 (gray thick dashes).
Stong biases, γ=0.1 and β=2 (black thick solid).

3. THE LABOR MARKET

We aim at estimating the impact for members of the minority group of the variations of the parameters which we assume are the source of network formation, on job arrival rate and unemployment. Heterogenity of networks come from differences in both average homophily rate and total number of contacts per individual for each type. These network characteristics have an impact on the way information on vacant job reaches on average individual of each type. We still have a stock $N = N_i + N_j$ of individuals and therefore the following relative sizes of population: n_i and $n_j = 1 - n_i$. Unlike I&S, we assume that ties are uniformly distributed, then each worker of type i have $s_i + d_i$ contacts with other individuals as well as each worker of type j have $s_j + d_j$ contacts with other individuals.

As I&S, we work on a discret time matching model *à la* Pissarides (2000). During a period - a day - individual may search for jobs through both formal methods (they can go to employment agencies, read newspaper, search through the web, etc.) and informal methods (they are embeded in social relationship and any of their direct employed contact may transmit information about vacant job). In fact we focus on the probability for an individual to receive a job offer taking into account both formal and informal method. At each period of time t_n, some workers will keep their job, others lose it and a part of unemployed individuals are hired. From each period t_{n-1}, $U_{t_{n-1}}$ unemployed individuals and $L_{t_{n-1}}$ employed individuals enter the next period t_n. We consider that all workers, whatever their type, have the same productivity. In order to simplify the mechanism, in the period they have been hired, workers'productivity is equal to y_0, then in the next period their productivity is equal to y_1. The same is true for wages with w_0 and w_1. This induces that employed worker have not any interest to keep a job offer for themselve. Finally, we consider that jobs and firms are identical, and that all individuals have the same skill, age, etc.

3.1. Job arrival rate at t_n

3.1.1. Formal method

We suppose that both employed individuals who search job for their friends and unemployed individuals have the same probability to receive a job offer when considering only formal method. We note $p(\theta_{t_n})$ or φ_{t_n} this probability to receive a job offer by a formal method at t_n. This probability is for both employed and unemployed

$$p(\theta_{t_n}) = \frac{M\left(U_{t_{n-1}} + \delta L_{t_{n-1}}, V_{t_n}\right)}{U_{t_{n-1}} + \delta L_{t_{n-1}}} = \varphi_{it_n} = \varphi_{jt_n} \tag{9}$$

where as in Fontaine (2008) δ can be considered as the percentage of employee who search a job for their friends, V_{t_n} is the number of vacant job in the economy at time t_n, $U_{t_{n-1}}$ is the number of unemployed entering the labor market at t_n such that $U_{t_{n-1}} = U_{it_{n-1}} + U_{jt_{n-1}}$, with $U_{it_{n-1}} = \left(1 - p_{it_{n-1}}\right) U_{it_{n-2}} + b\left(N_i - U_{it_{n-2}}\right)$ the number of unemployed individual of type i and $U_{jt_{n-1}} = \left(1 - p_{jt_{n-1}}\right) U_{jt_{n-2}} + b\left(N_j - U_{jt_{n-2}}\right)$ the number of unemployed individual of type j, where b is the job destruction rate at each period and p the probability to get a job either through formal and informal method. We find after some simplification that

$$U_{t_{n-1}} = \left(1 - p_{it_{n-1}}\right) U_{it_{n-2}} + \left(1 - p_{jt_{n-1}}\right) U_{jt_{n-2}} + bL_{t_{n-2}} \tag{10}$$

and

$$L_{t_{n-1}} = N - U_{t_{n-1}} \tag{11}$$

Moreover, we note $\theta_{t_n} = \frac{V_{t_n}}{U_{t_{n-1}}+\delta L_{t_{n-1}}}$ the labor market tightness at t_n.

3.1.2. Informal method

Whatever the period t_n, we respectively note u_i and u_j the unemployment rate of individuals of type i and type j. Unlike I&S and Calvò-Armengol & Zenou (2005), because we have two types of agents, in order to simplify the model, we assume that unemployment is uniformly distributed accross networks. Indeed, in our model, each individual of type i has $s_i u_i$ unemployed contacts of the same type and $s_i(1-u_i)$ employed contacts of the same types. In the same way each individual of type i has $d_i u_j$ unemployed contacts of the other type and $d_i(1-u_j)$ employed contacts of the other type. The same is true for individual of type j. This may appear as an important simplification but, since the number of contacts each agent gets tend to be important, it does not greatly change the results[2].

However, probability for an unemployed individual of type i to get a job through network is not the same when we consider contacts of type i and contacts of type j. Since an individual of type i is unemployed, he has one chance out of $1+(s_i-1)u_i+d_i u_j$ that one of his contact of the same type choose him through his unemployed contacts. In the same way, an individual of type j have one chance out of $u_j s_j+1+(d_j-1)u_i$ to be chosen through unemployed contact of one of his contact of type j. Then, probability for an individual of type i that an employed individual of the same type who has received and transmitted a job offer, when this unemployed individual has $s_i(1-u_i)$ type i employed contacts, is $\delta \times \frac{1}{1+(s_i-1)u_i+d_i u_j} \times s_i(1-u_i)$. Remember that δ is the probability that an employed individual transmit the job offer.

Knowing this and the fact that an employed individual receive a job offer at rate $p(\theta)$, we deduce the probability $\widetilde{\varphi}_{i-i}$ that an unemployed individual of type i receive a job offer through a contact of the same type i:

$$\widetilde{\varphi}_{i-i} = p(\theta) \times \delta \times \frac{1}{1+(s_i-1)u_i+d_i u_j} \times s_i(1-u_i) \tag{12}$$

In the same way we find the probability $\widetilde{\varphi}_{i-j}$ for type i to receive a job offer through a contact of type j:

$$\widetilde{\varphi}_{i-j} = p(\theta) \times \delta \times \frac{1}{u_j s_j+1+(d_j-1)u_i} \times d_i(1-u_j) \tag{13}$$

[2] Our result tend to the one of Calvò-Armengol and Zenou (2005) when the number of contacts an individual has tends to infinity. Calvò-Armengol and Zenou (2005) use a binomial ditribution to decribed the distribution of unemployment across networks.

Each individual of type i have then a probability $\widetilde{\varphi}_i = \widetilde{\varphi}_{i-i} + \widetilde{\varphi}_{i-j}$ to receive a job offer through his SN, therefore after simplification we have:

$$\widetilde{\varphi}_i = p(\theta) \times \delta \times \left(\frac{s_i (1 - u_i)}{1 + (s_i - 1) u_i + d_i u_j} + \frac{d_i (1 - u_j)}{u_j s_j + 1 + (d_j - 1) u_i} \right) \tag{14}$$

3.1.3. Formal method + informal method

Probability p_i to get a job for an individual of type i is

$$p_i = p(\theta) + \widetilde{\varphi}_i$$

then after simplification

$$p_i = p(\theta) \left(1 + \delta \times \left(\frac{s_i (1 - u_i)}{1 + (s_i - 1) u_i + d_i u_j} + \frac{d_i (1 - u_j)}{u_j s_j + 1 + (d_j - 1) u_i} \right) \right) \tag{15}$$

and

$$p_j = p(\theta) \left(1 + \delta \times \left(\frac{s_j (1 - u_j)}{1 + (s_j - 1) u_j + d_j u_i} + \frac{d_j (1 - u_i)}{s_i u_i + 1 + (d_i - 1) u_j} \right) \right) \tag{16}$$

3.2. Job filling rate for firms

On the one hand, the rate at which a firm fill a vacant job, when we take into account only formal method, is $\frac{M(u+\delta(1-u),v)}{v} = \frac{p(\theta)}{\theta}$.

On the other hand the rate at which a firm fill a vacant job with type i when we take into account only informal method is $\frac{p(\theta)}{\theta} \frac{n_i(1-u_i)}{u+\delta(1-u)} \times \delta \times \frac{s_i u_i}{s_i u_i + d_i u_j} + \frac{p(\theta)}{\theta} \frac{n_j(1-u_j)}{u+\delta(1-u)} \times \delta \times \frac{d_j u_i}{s_j u_j + d_j u_i}$ where $\frac{n_i(1-u_i)}{1-u}$ is the probability for a type i employed individual to be reached, δ is the probability that he transmits the job offer to one of his unemployed contact, $\frac{s_i u_i}{s_i u_i + d_i u_j}$ is the probability that he choose a contact of type i and $\frac{n_j(1-u_j)}{1-u} \times \delta \times \frac{d_j u_i}{s_j u_j + d_j u_i}$ is the probability for a type i unemployed individual to be reached by an employed individual of type j of his network.

Finally, a firm fill a vacant job with a type i unemployed individual with probability $f_i = \frac{n_i u_i}{u+\delta(1-u)} \frac{p(\theta)}{\theta} + \frac{p(\theta)}{\theta} \frac{\delta n_i(1-u_i)}{u+\delta(1-u)} \times \frac{s_i u_i}{s_i u_i + d_i u_j} + \frac{p(\theta)}{\theta} \frac{\delta n_j(1-u_j)}{u+\delta(1-u)} \times \frac{d_j u_i}{s_j u_j + d_j u_i}$, then

$$f_i = \frac{p(\theta)}{\theta} \left(\frac{n_i u_i}{u + \delta (1 - u)} + \delta \left(\frac{n_i (1 - u_i)}{u + \delta (1 - u)} \frac{s_i u_i}{s_i u_i + d_i u_j} + \frac{(1 - n_i)(1 - u_j)}{u + \delta (1 - u)} \frac{d_j u_i}{s_j u_j + d_j u_i} \right) \right)$$

and the total probability to fill a vacant job whatever the type of the individual, is

$$f = f_i + f_j$$

3.3. The steady-state unemployment rate

During each period t_n, a proportion of type i unemployed individuals find a job with probability p_{it_n}. A rate $u_{it_{n-1}}p_{it_n}$ of individual of type i thus enters employment at the begining of t_n, where $u_{it_{n-1}}$ is the unemployment rate of a type i individual when entering t_n. Moreover, we have a rate $1 - u_{it_{n-1}}$ of type i employed individuals at the begining of t_n. If, at each period, some employed individuals lose their job at rate b, individual of type i lose their job at t_n at rate $b\left(\left(1 - u_{it_{n-1}}\right) + u_{it_{n-1}}p_{it_n}\right)$.

Evolution of unemployment between t_n and t_{n-1} is equal to the difference between those who have lost their job at the end of t_n and those who enter employment a the begining of t_n , then

$$u_{it_n} - u_{it_{n-1}} = b\left(\left(1 - u_{it_{n-1}}\right) + u_{it_{n-1}}p_{it_n}\right) - u_{it_{n-1}}p_{it_n}$$

We then have, at the steady state, with $u_{it_n} = u_{it_{n-1}} = u_i$, and $p_{it_n} = p_{it_{n+1}} = p_i$

$$u_i = \frac{b}{p_i\left(1-b\right)+b} \tag{17}$$

We also find that

$$u_j = \frac{b}{\left(2p(\theta)\left(1-b\right)+b\right)\left(1-n_i\right)} + \frac{u_i n_i}{1-n_i} \tag{18}$$

and that unemployment rate in the whole economy at the steady state is

$$u = \frac{b}{\left(1-b\right)p+b} \tag{19}$$

3.4. Workers utility, profits and wages

3.4.1. Utility of employed and unemployed individuals

Utility of unemployed individuals

If $W_{U_{it_n}}$ is type i unemployed individual utility, r is the real interest rate, w_0 is the wage of an individual entering employment and $W_{E_{it_{n+1}}}$ the utilty of a type i employed individual at t_{n+1}, then

$$W_{U_{it_n}} = \frac{1}{1+r}\left(1-p_{it_n}\right)W_{U_{it_{n+1}}} + p_{it_n}\left[w_0 + \frac{1}{1+r}\left(\left(1-b\right)W_{E_{it_{n+1}}} + bW_{U_{it_{n+1}}}\right)\right]$$

At the steady state, $W_{E_{it_n}} = W_{E_{it_{n+1}}} = W_{E_i}$ and $W_{U_{it_n}} = W_{U_{it_n}} = W_{U_i}$. We finally have

$$W_{U_i} = \frac{1}{1+r}\left(1-p_i\right)W_{U_i} + p_i\left[w_0 + \frac{1}{1+r}\left(\left(1-b\right)W_{E_i} + bW_{U_i}\right)\right] \tag{20}$$

Utility of employed individuals

Utility $W_{E_{it_n}}$ of a type i employed individual is:

$$W_{E_{it_n}} = w_{1_i} + \frac{1}{1+r}\left[\left(1-b\right)W_{E_{it_{n+1}}} + bW_{U_{it_{n+1}}}\right]$$

then at the steady state

$$W_{E_i} = w_{1_i} + \frac{1}{1+r}\left[\left(1-b\right)W_{E_i} + bW_{U_i}\right] \tag{21}$$

with w_1 the wage and b the job destruction rate.

3.4.2. *The expected profit for firms*

The expected profit $\Pi_{E_{it_n}}$ of a filled job by a worker of type i who have on average $s_i + d_i$ contacts is at t_n:

$$\Pi_{E_{it_n}} = y_1 - w_{1_i} + \frac{1}{1+r}\left[(1-b)\Pi_{E_{it_{n+1}}} + b\Pi_{V_{t_{n+1}}}\right] \tag{22}$$

where y_1 is the productivity of an employed individual, w_{1_i} is the wage of an individual of type i and $\Pi_{V_{t_{n+1}}}$ is the expected profit of a vacant job for a firm at t_{n+1}. Moreover, if f is the rate at which a firm fill a vacant job and $\frac{f_i}{f}$ the probability to fill a job through a type i unemployed individual, then $\Pi_{V_{t_n}}$ is:

$$\Pi_{V_{t_n}} = -h + (1-f)\frac{1}{1+r}\Pi_{V_{t_{n+1}}} + f\left[y_0 - w_0 + \frac{1}{1+r}\left((1-b)\times\frac{f_i}{f}\times\Pi_{E_{it_{n+1}}} + b\Pi_{V_{t_{n+1}}}\right)\right] \tag{23}$$

3.4.3. *Free entry condition at the steady state.*

At the steady state we have $\Pi_{E_{it_n}} = \Pi_{E_{it_{n+1}}} = \Pi_{E_i}$ and $\Pi_{V_{t_n}} = \Pi_{V_{t_{n+1}}} = \Pi_V$. If $y_0 = w_0 = 0$ and $\Pi_V = 0$ we then have

$$\Pi_{E_i} = \frac{\left(y_1 - w_{1_i}\right)\left(1+r\right)}{r+b} \tag{24}$$

In the same way we find

$$\Pi_{E_i} \times f_i = \frac{1+r}{1-b}h \tag{25}$$

from which we have the free entry condition

$$(y_1 - w_{1_i}) \times f_i = \frac{r+b}{1-b} h \tag{26}$$

3.4.4. *Wage determination at the steady state*

From (21) and (20) we find

$$W_{E_i} = w_{1_i} + \frac{1}{1+r} \left[(1-b) W_{E_i} + b W_{U_i} \right]$$

$$W_{U_i} = \frac{1}{1+r} (1 - p_i) W_{U_i} + p_i \left[w_0 + \frac{1}{1+r} \left((1-b) W_{E_i} + b W_{U_i} \right) \right]$$

We then have at the steady state

$$W_{E_i} - W_{U_i} = \frac{1+r}{r + b + p_i (1-b)} w_{1_i} \tag{27}$$

If we note $x \in (0,1)$ the surplus share attributed to workers and if wages are Nash bargaining we have

$$w_{1_i} = \arg\max \left(W_{E_i} - W_{U_i} \right)^x \left(\Pi_{E_i} - \Pi_V \right)^{1-x}$$

We then have the following first order condition

$$(1-x) \left(W_{E_i} - W_{U_i} \right) = x \left(\Pi_{E_i} - \Pi_V \right) \tag{28}$$

From (28), (27) and (25) and from the free entry condition $\Pi_V = 0$,

$$(1-x) \left(W_{E_i} - W_{U_i} \right) = x \left(\Pi_{E_i} - \Pi_V \right)$$

$\iff$

$$w_{1_i} = \frac{x (r+b) + x p_i (1-b)}{r + b + x p_i (1-b)} y_1 \tag{29}$$

In the same way we find

$$w_{1j} = \frac{x (r+b) + x p_j (1-b)}{r + b + x p_j (1-b)} y_1 \tag{30}$$

4. CALIBRATION AND RESULTS INTERPRETATION

In order to calibrate our model for types i and j, we have the following system (see appendix for more details about the choice of parameters chosen for calibration):

$$
\begin{cases}
(y_1 - w_{1_i}) \times f_i = \frac{r+b}{1-b} h \ (26) \\
W_{U_i} = \frac{1}{1+r} (1 - p_i) W_{U_i} + p_i \left[w_0 + \frac{1}{1+r} ((1-b) W_{E_i} + b W_{U_i}) \right] \ (20) \\
W_{E_i} = w_{1_i} + \frac{1}{1+r} [(1-b) W_{E_i} + b W_{U_i}] \ (21) \\
\Pi_{E_i} \times f_i = \frac{1+r}{1-b} h \ (25) \\
u_i = \frac{b}{p_i (1-b) + b} \ (17) \\
u_j = \frac{b}{(2p(\theta)(1-b)+b)(1-n_i)} + \frac{u_i n_i}{1-n_i} \ (18) \\
u = \frac{b}{(1-b)p + b} \ (19) \\
w_{1_i} = \frac{x(r+b) + x p_i (1-b)}{r+b+x p_i (1-b)} y_1 \ (29) \\
p_i = p(\theta) \left(1 + \delta \times \left(\frac{s_i (1-u_i)}{1+(s_i-1)u_i + d_i u_j} + \frac{d_i (1-u_j)}{u_j s_j + 1 + (d_j - 1) u_i} \right) \right) \ (15) \\
p_j = p(\theta) \left(1 + \delta \times \left(\frac{s_j (1-u_j)}{1+(s_j-1)u_j + d_j u_i} + \frac{d_j (1-u_i)}{s_i u_i + 1 + (d_i - 1) u_j} \right) \right) \ (16)
\end{cases}
$$

4.1. The effect of variations in n_i, γ and β on both job arrival rate and unemployment rate

We choose a set of value for γ and β in order to determine the impact of a variation in both preferences and meeting opportunities biases on both the job arrival rate and the unemployment rate.

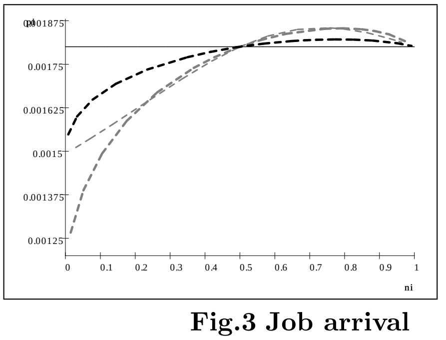

Fig.3 Job arrival rate $p_i(n_i)$.

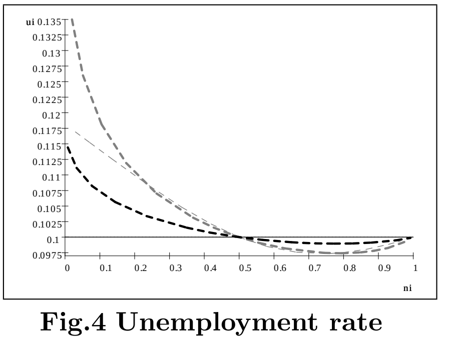

Fig.4 Unemployment rate $u_i(n_i)$**.**

No biases, $\gamma = 1$ and $\beta = 1$ (black thin solid).
Weak biases, $\gamma = 0.5$ and $\beta = 2$ (black thick dashes).
Medium biases in preferences and no biases in meeting opportunities, $\gamma = 0.5$ and $\beta = 1$ (gray thin dashes).
Strong biases in preferences, $\gamma = 0.1$ and $\beta = 2$ (gray thick dashes).

4.1.1. The main observations

From Fig.3, when there are neither biases in meetings nor biases in preferences (γ and β are equal to 1), networks have the same impact for the two groups (see the black thin solid). Individuals of each type have the same number of contact $l_i = l_j$ and have the same probability to get a job whatever n_i and n_j.

What about the influence of variations in γ and β when $\gamma < 1$ and $\beta > 1$? A rise in biases in preferences ($\gamma \to 0$) has a negative impact on employment for the minority when β is fixed (move from the black thick dashed curve to the gray thick dashed curve). The effect is positive for the majority. On the contrary, when γ is fixed, a rise in biases in meeting opportunity ($\beta \to \infty$) have a positive impact on employment for the minority (move from the gray thin dash curve to the black thick dash curve). The effect is negative for the majority.

We have to explain with more details the mecanisms. In order to do this, we can observe what is happening in both Fig.1 and Fig.2, where we see how variations in both γ and β influence both $l_i(n_i)$ and $H_i(n_i)$. We must then understand how variations of $l_i(n_i)$ and $H_i(n_i)$ affect $p_i(n_i)$ and $u_i(n_i)$.

4.1.2. Explanation of the impact of variations in γ on $p_i(n_i)$ and $u_i(n_i)$ when β is fixed

On the one hand preferences and biases in meeting opportunities influence $l_i(n_i)$. When β is constant, (see the curves where $\beta = 2$), a rise in preferences (γ goes from 0.5 to 0.1 for instance) have a negative impact on $l_i(n_i)$, for the majority as for the minority (see Fig.2). The fact that an individual has less contacts has a negative effect on his probability to receive a job offer through SN. Then a rise in preferences have a negative impact on unemployment ($u_i(n_i)$ is rising). But in the other hand preferences and biases in meeting opportunities have an impact on $H_i(n_i)$ which influences indirectly the probability to receive a job offer through SN. Still when β is constant, a rise in preferences (γ goes from 0.5 to 0.1) has a weak negative impact on $H_i(n_i)$ for the minority but weakly positive for the majority (see Fig.1). Yet, if $u_i(n_i)$ rises when γ goes from 0.5 to 0.1 with a constant β, having less contact of type i (which is the case for type i when type i is the minority) lowers the partly negative effect of the rise of $l_i(n_i)$ on $p_i(n_i)$ and $u_i(n_i)$ (type i have a higher unemployment rate than type j so it is better to have contact of type j to improve probability to receive a job offer through SN). The effect on H_i being weak, we finally observe on Fig.3 and 4 when $\gamma \to 0$, with a constant β, that the global effect on employment for minority is negative (the effect of $l_i(n_i)$ on $p_i(n_i)$ and $u_i(n_i)$ prevail on the effect of $H_i(n_i)$ on $p_i(n_i)$ and $u_i(n_i)$).

4.1.3. Explanation of the impact of variations in β on $p_i(n_i)$ and $u_i(n_i)$ when γ is fixed

We can have similar interpertation for the impact of a variation in β (for instance β goes from 1 to 2) with γ fixed ($\gamma = 0.5$) on $p_i(n_i)$ and $u_i(n_i)$. The number of contact $l_i(n_i)$ is rising, which have a positive impact on employment. Homophily rate $H_i(n_i)$ being rising, having more contacts of type i have this time a positive impact on employment.

4.1.4. Explanation of the shape of the curves $p_i(n_i)$ and $u_i(n_i)$

When biases exists, we observe that probability to receive job offer are always rising with the relative size of the population n_i untill some value of n_i. This increasing and decreasing form is shared by all biased cases (when $0 \leq \gamma < 1$ et $\beta > 1$). This could be easily explained. As in Calvò-Armengol and Zenou (2005) probability to receive a job offer is rising with the size of networks of each individuals (we easily verify that l_i is strictly rising with n_i). Then, from some value of l_i the probability to receive job offer decreases because of a congestion effect (for a giving number of unemployed individual in the network the rise of competition between unemployed have a negative effect on $p_i(n_i)$). Moreover, the marginal decrease of $p_i(n_i)$ could partly be explained through the marginal decrease of l_i with n_i (see Fig.2).

About unemployment, when there are no biases ($\gamma = 1$ and $\beta = 1$), we find the exogenous global 10% rate we have fixed for both the minority and the majority (horizontal black thin solid in Fig.4). Since preferences exist, unemployment curves became convex. When biases are identical for the two groups, unemployment rate are higher for the minority than the average unemployment rate, conversely for the majority. Moreover, we remark that SN affect as much the difference between the unemployment rate of the minority and the global 10% rate when the relative size of the minority is small.

4.2. The impact of variations in γ and β on inequalities between groups

We have seen how variation of n_i, γ et β influence $p_i(n_i)$ and $u_i(n_i)$. Let's now interprete the influence of the variation of n_i, γ et β on inequalities between the majority and the minority when the global unemployment rate is fixed to 10%. We will fromnow always consider i as the minority and j as the majority, then $n_i < n_j$. All the graph from Fig.5 can be read as follow. When $n_i = 0.1$ we must deduce that $n_j = 1 - n_i = 0.9$. Probability for the majority must be reading from the right to the left with $n_j \in [0.5, 1]$. For each point (n_i, n_j) we can compare the value (p_i, p_j) and (u_i, u_j) or the differences$(p_j - p_i)$ and $(u_j - u_i)$.

4.2.1. Medium biases

On the Fig.5, probability to receive a job offer for an individual of type i is widely modified as compared with the case without biases. Inequality is rising with the differences $n_j - n_i$, whatever the value of $\gamma < 1$ and $\beta > 1$. For the value of the parameters we have chosen ($\gamma = 0.5$ et $\beta = 2$), inequalities are not higher than 1.5 points of % (Fig.6).

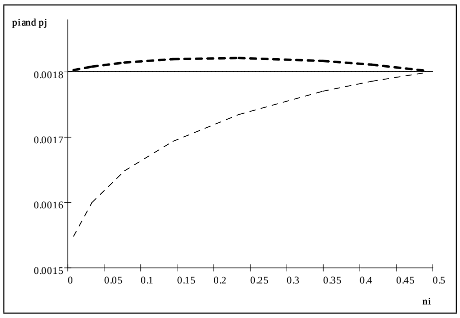

Fig.5 Job arrival rates $p_i(n_i)$ et $p_j(n_i)$**.**

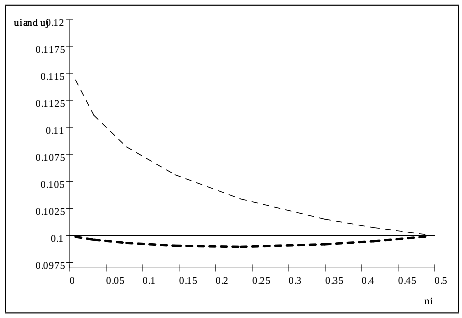

Fig.6 Unemployment rate $u_i(n_i)$ et $u_j(n_i)$**.**

No biases, $\gamma = 1$ and $\beta = 1$ (black thin solid).
Medium biases, $\gamma = 0.5$ and $\beta = 2$ (minority $\rightarrow$ thin dashes).

4.2.2. Medium biases in preferences and no biases in meetings

When individuals have preferences ($\gamma \in [0, 1[$) but when they meet at random ($\beta = 1$, Fig.7 et 8), job arrival rate is no more a concave function of n_i for the minority group.

About unemployment, in comparing Fig.8 with Fig.6, we see that meeting biases lowered inequalities between the two groups. The gap stay quite large untill relative size became very close. In other words, biases in meeting opportunities reduce the penalty for the minority group.

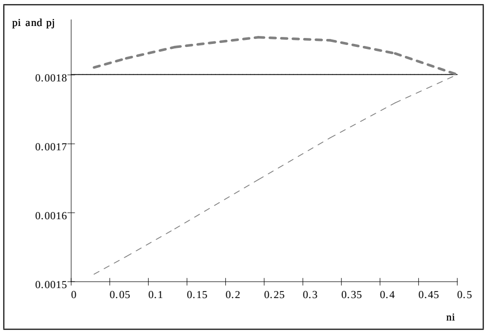

Fig.7 Job arrival rates $p_i\,(n_i)$ et $p_j\,(n_i)$**.**

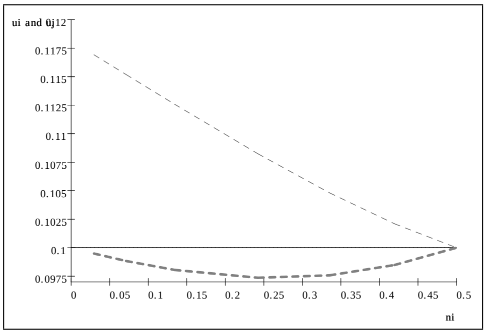

Fig.8 Unemployment rates $u_i\,(n_i)$ et $u_j\,(n_i)$**.**

No biases, $\gamma = 1$ and $\beta = 1$ (black thin solid).
Medium biases in preferences and no meeting biases,
$\gamma = 0.5$ and $\beta = 1$ (minority $\rightarrow$ thin dashes).

4.2.3. Strong preferences and medium biases in meeting opportunities

When comparing Fig.5 and Fig.9 we see that a strong rise in preferences increases significantly inequalities between the two groups. In the Fig.10, we see that the gap between the two groups reach 4% which is much higher than in the Fig.6.

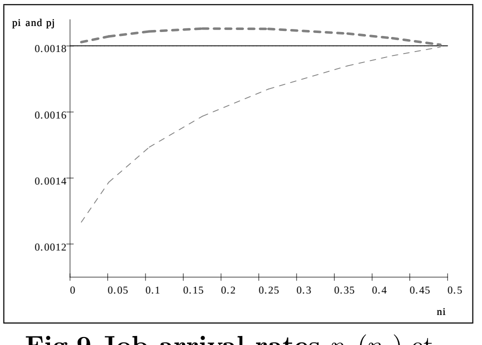

Fig.9 Job arrival rates $p_i\,(n_i)$ et $p_j\,(n_i)$**.**

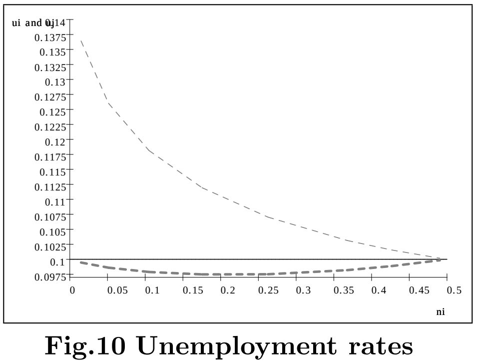

Fig.10 Unemployment rates $u_i\,(n_i)$ et $u_j\,(n_i)$**.**

No biases, $\gamma = 1$ and $\beta = 1$ (black thin solid).
Strong preferences and medium biases in meeting opportunities,
$\gamma = 0.1$ and $\beta = 2$ (minority $\rightarrow$ thin dashes).

4.3. When the minority have higher biases in meetings ($\beta_i >> \beta_j$).

We saw that a rise in meeting opportunity biases for the minority lower the least efficiency of their SN in getting job offer. When the minority is better organized than the majority, in term of sociability, or when ethnic urban segregation is for instance higher for the minority like in the empirical studies we mentionned above, unemployment rate for the minority can be less important than unemployment rate of the majority for some giving value of n_i (Fig.11).

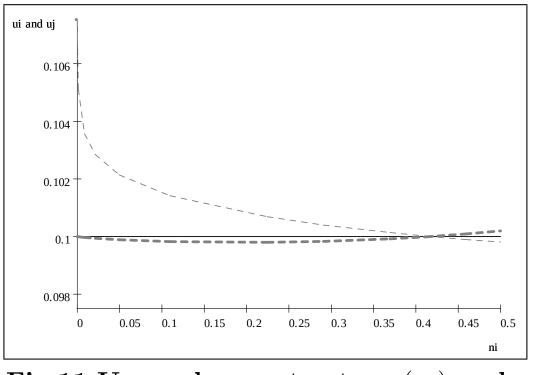

Fig.11 Unemployment rate $u_i(n_i)$ and $u_j(n_i)$.

No biases, γ=1 et β=1 (black thin solid).
Medium preferences, γ=0.5 for i and j.
Strong biases in meeting opportunities for i, β_i=8 (gray thin dashes).
Medium biases in meeting opportunities for j, β_j=2 (gray thick dashes).

5. THE EFFECT OF SN + HIRING PENALTY ON BOTH JOB ARRIVAL RATE AND UNEMPLOYMENT RATE FOR THE MINORITY GROUP

In the previous part, we have isolated the effect of SN on $p_i(n_i)$ and $u_i(n_i)$ and particularly the effect of n_i, γ and β on $p_i(n_i)$ and $u_i(n_i)$. Yet, for all pratical puposes, it is possible that hiring discrimination for minority coexist with SN and that the two phenomenon influence each other. Have n_i, γ and β the same effect on job arrival rates and unemployment rates when the minority is subjected to hiring discrimination?

5.1. $p_i(n_i)$ and $u_i(n_i)$ when D exist

We just add hiring discrimination to the previous model for the minority group. An unemployed individual of type i, for $n_i \in [0, 0.5]$, have a probability $Dp(\theta)$ to receive a job offer through a formal method where $D \in [0,1]$ could in fact be any hiring penalty (hiring discrimination from employer, spatial mismatch, etc.). D represents the facts that information flows through formal method is now weaker for unemployed individuals from the minority group than unemployed individuals from the majority group. An unemployed individual of type j has then a probability $p(\theta)$ to receive a job offer. In the same way, we consider that a type i unemployed individual has a probability $D\widetilde{\varphi}_i$ to receive a job offer through informal method beacause he would have to meet employer after receiving the job offer. A type j unemployed individual will receive a job offer through informal method with probability $\widetilde{\varphi}_j$. Unlike the case without discrimination, an unemployed individual from the minority is no more automatically hired when he receives a job offer. Specify just that employed individual of the two types will still have the same probability to receive a job. We then have

$$p_i = Dp(\theta)\left(1+\delta\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j}+\frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right)\right)$$

5.2. The effect of variations in n_i, γ and β on $p_i(n_i)$ with D

5.2.1. The main observations

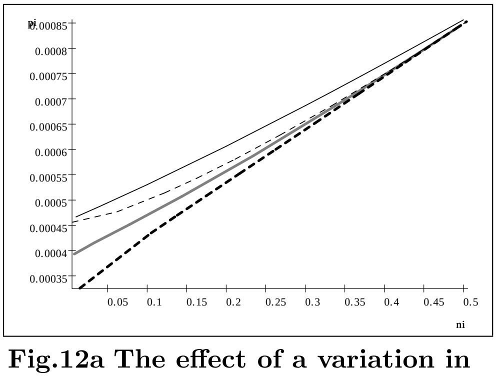

Fig.12a The effect of a variation in γ on $p_i(n_i)$.

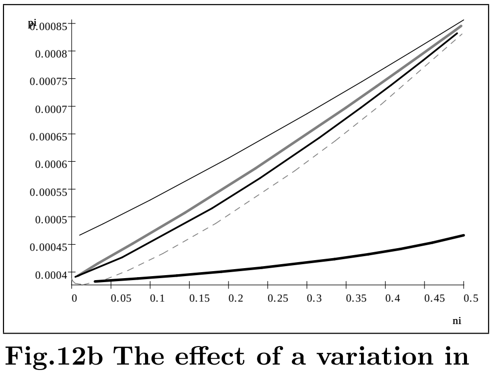

Fig.12b The effect of a variation in β on $p_i(n_i)$.

No biases, γ=1 and β=1(black thin solid) → 12a and 12b.
Medium pref. and biases in meetings, γ=0.5 and β=2 (gray thick solid) → 12a and 12b.
Strong pref. and medium biases in meetings, γ=0.1 and β=2 (black thick dashes) → 12a.
No pref. and medium biases in meetings, γ=1 and β=2 (black thin dashes) → 12a.
Medium pref. and no biases in meetings, γ=0.5 and β=1 (black thick solid) → 12b.
Medium pref. and strong biases in meetings, γ=0.5 and β=3 (black medium solid) → 12b.
Medium pref. and very strong biases in meetings, γ=0.5 and β=5 (gray thin dashes) → 12b.

Hiring discrimination modify the effect of SN for minority. This is what we observe in comparing Fig.3, Fig.12a and Fig.12b. Whatever biases ($0 \leq \gamma \leq 1$ et $\beta \geq 1$), $p_i(n_i)$ is still increasing in n_i for minority but conversely to the case without discrimination where the representative curve of $p_i(n_i)$ was concave, the curve is now slightly convex in most cases. Moreover, this probability is now increasing in n_i even without biases. The fact that one have more or less contacts from the minority in his contacts (we still suppose that type i is the minority) have this time a direct impact on employment.

Before giving more explaination, we note that biases do not have the same effect as in the case without discrimination. A rise in preferences has the same effect as before as we see in the Fig.12a and this effect, with a fixed β, still diminishes with the rise of n_i. Instead of the case without discrimination, the fact that bias in meeting opporunity exist when preferences do not exist have an impact on employement. Biases in meeting opportunity have a negative impact on $p_i(n_i)$ for members of the minority.

We observe in the Fig.12b, when biases in meeting opportunity increase, if preferences exist, that the effect has changed as compared to the case without discrimination. Indeed when $\beta > 1$, a rise in β have a negative impact on employment. But the fact that biases in meeting opportunities exist have a positive effect on $p_i(n_i)$ for the minority.

5.2.2. Explanation of the impact of variations in n_i, γ and β on $p_i(n_i)$

The variation of $H_i(n_i)$ and $l_i(n_i)$ that we observe on Fig.1 et Fig.2 help us to interprete the previous observations. Why does the rise in γ when β is fixed still have a negative impact on $p_i(n_i)$? The fact that a rise in γ have a negative impact on l_i is still valid. But what is different from the case with discrimination is that having a biased $H_i(n_i)$ (more contacts of type i than at random) have a direct negative impact on $p_i(n_i)$. Indeed, individuals from the minority have a lower probability to receive a job offer through formal method when they are unemployed. Then it is better to have more contacts from the majority than from the minority. That is why biases in meeting opportunities now have a negative impact on $p_i(n_i)$ when there is no preferences. This is also why a rise in β when γ is constant has a negative impact on $p_i(n_i)$. If the number of contacts is rising, this time the fact that an individual have more contacts from the minority prevail.

5.3. The effect of variations in n_i, γ and β on $u_i(n_i)$ with D

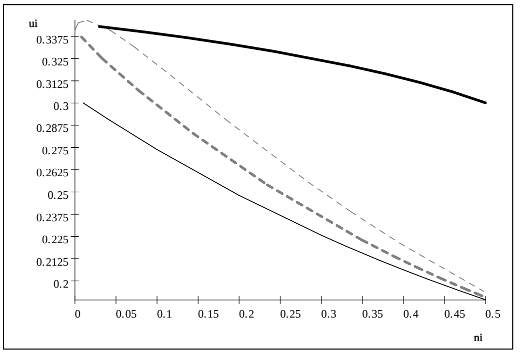

Fig.13 Unemployment rate for the minority $u_i(n_i)$.

No biases, γ=1 and β=1 (black thin solid).
Medium preferences and biases in meeting opportunities, γ=0.5 and β=2 (gray thick dashes).
Medium preferences and strong bias in meeting opportunities, γ=0.5 and β=5 (gray thin dashes).
Medium preferences and no biases in meeting opportunities, γ=0.5 and β=1 (black thick solid).

If the form of the probability curves is modified, the form of unemployment curves is still convex even if convexity have been lowered by hiring discrimination.

About the variations of γ and β, we find the same effect as for $p_i(n_i)$. We see on the Fig.13 that discrimination influences much more unemployment rates than SN because with $D = 0.5$, unemployment rate is much more higher than in the case without discrimination.

5.4. Does hiring discrimination modify the effect of SN on inequalities?

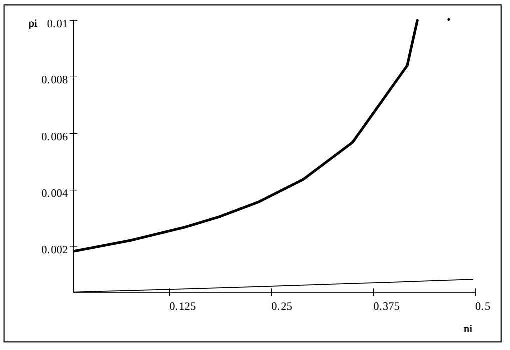

Fig.14 $p_i(n_i)$ **with and without discriminations**.

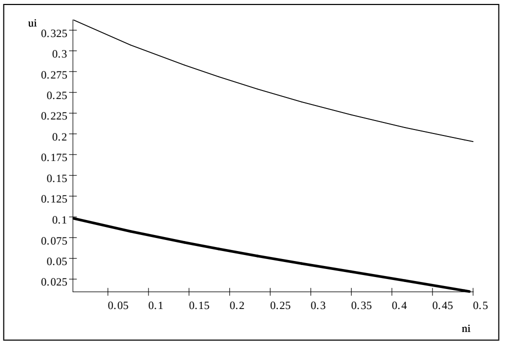

Fig.15 Unemployment rate $u_i(n_i)$ **with and without discrimination**.

Medium biases, γ=0.5 et β=2.
No discriminations → black thick solid.
With discriminations → black thin solid.

Comparison of Fig.5 and 14 shows that hiring discrimination for minority transforms the effect of SN on inequalities. Indeed, when there were no hiring discrimination, inequalities were decreasing in n_i. This is no more the case when hiring discrimination exists, inequalities rise with n_i (see Fig.15).

5.5. Comparison of the effect of SN on $p_i(n_i)$ and $u_i(n_i)$ with and without hiring discrimination

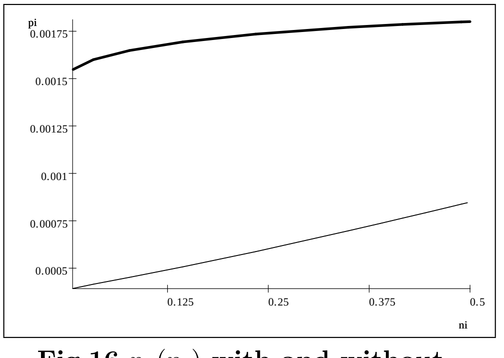

Fig.16 $p_i(n_i)$ **with and without discriminations**.

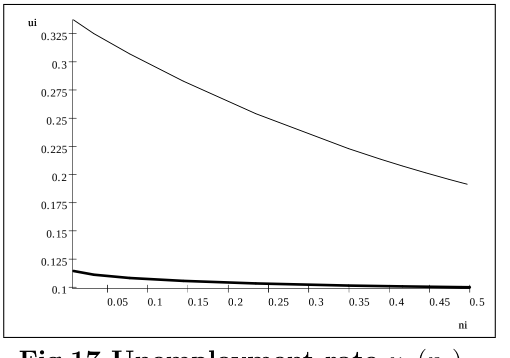

Fig.17 Unemployment rate $u_i(n_i)$ **with and without discriminations**.

Medium biases, γ=0.5 and β=2.
No discriminations → black thick solid.
With discriminations → black thin solid.

The rise of the positive effect of networks with n_i is higher when hiring discrimination exist. Indeed, we observe on Fig.16 that the rise of n_i has a bigger impact on $p_i(n_i)$ with discrimination than on $p_i(n_i)$ without discrimination. The difference between the two curves is thightening when n_i rises. This phenomenon also appears in the Fig.17. Finally, although hiring discrimination is an additional penalty for minority, the impact of the rise of $l_i(n_i)$ on labor market outcomes is higher when discrimination exists.

5.6. What happens when the minority have higher biases in meeting opportunity ($\beta_i >> \beta_j$)?

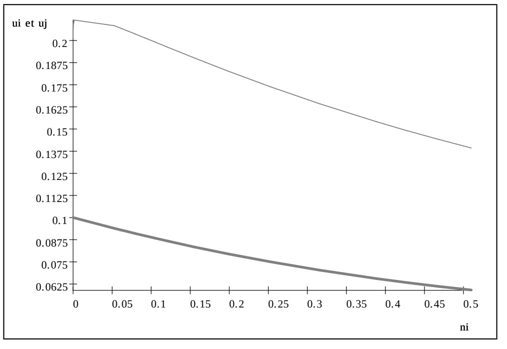

Fig.18 Unemployment rate $u_i(n_i)$ and $u_j(n_i)$.

Medium preferences, γ=0.5 for i and j.
Very strong biases in meeting opportunities for the minority, β_i=20 (gray thin dashes).
Medium biases in meeting opportunity for the majority, β_j=2 (gray thick dashes).

We do not find anymore the intersection between the two curves as in the Fig.11, even with a very strong biases in meeting opportunity for the minority ($\beta_i = 20$).

6. CONCLUDING REMARKS

We found that determinants of networks formation have an impact on both $H_i(n_i)$ (the proportion of individual of the same type i in the network of an individual of type i) and $l_i(n_i)$ (the total number of contacts an individual of type i has), and that these two elements have therefore an impact on employment for individuals from the minority group since they have same-type ties preferences. A rise in $l_i(n_i)$ has a positive impact on employment for members of the minority group whereas a rise in $H_i(n_i)$ has a negative impact on employment for members of this group. We then show that a rise in the attachment to the culture of origin has a negative impact on $l_i(n_i)$ and a positive impact on $H_i(n_i)$. In this case there are two negative impacts on employment, therefore a global negative impact on employment for members of the minority group. A rise in biases in meetings have a positive impact on both $l_i(n_i)$ and $H_i(n_i)$. In this case, there are at the same time a positive and a negative impact on employment for the minority. When hiring discrimination does not exist, the positive effect of $l_i(n_i)$ is higher. When hiring discrimination for the minority exists, the negative effect of $H_i(n_i)$ is higher.

Before mentioning some of the limits of our model, two remarks may be made. First, unlike the empical models we mentioned in the introduction, we offer a complete theoretical framework to better understand the impact of the determinant of social contacts on labor market outcomes for minority groups. For instance, Domingues Dos Santos (2005) has shown that SN are not as efficient for members of the Portuguese community and members of the North African community in France but do not offer any explanation. Thanks to our model, we can imagine that either lower biases in meeting opportunities or conversely higher biases in meeting opportunities coupled with hiring discrimination, in both cases for North African individuals, have on average a negative impact on the effect of their SN (see Fig.11 and Fig.18). In order to be more rigorous and to choose between these two assumptions, this should of course be empirically studied. The second remark is that our theoretical framework which takes into account at the same time many parameters of network formation allows us to make better political recommendation than the empirical studies we above-mentioned which separatly study the effect of one parameter. Unlike what insinuate for instance the results of Patacchini et Zenou (2008), improving urban ethnic segregation won't always have a positive impact on employment for individuals from the minority group. Our model shows that this is only true in some cases, when both attachment to culture of origin exists and hiring discrimination does not exist. Moreover, our results allow us to say that policies which lower preferences for same-type ties are always better than policies which improve biases in meeting opporunitiies when we aim at diminishing inequalities between members of minority and majority groups.

Finally, we must mention some limits of our model. First, we make the assumption that the probability δ that an employed individual transmits a job offer is the same for the minority and the majority. But one can imagine that transmitting a job offer could be more useful for individuals from the minority because they will probably need it more than members of the majority in the future, all the more so if they are subjected to hiring discrimination. In fact solidarity would maybe be higher between members of the minority group. This may have an impact on the results of our model and should be integrated in future researches. Another remark should be made about the fact that we fix the global unemployment rate u. This simplifies our calculations and make our model more realistic in incorporating some institutionnal specificities but it restrains variations in p_i and p_j (the same for u_i and u_j). This is not important when we study the impact of variations in $l_i(n_i)$ and $H_i(n_i)$ on the sense of the variation of the probability to receive a job offer, the unemployment rates and inequalities between the minority and the majority groups. But we cannot make a real judgment on the size of the variation of the probability to receive a job offer, on the size of the variation in both the unemployment rate and the size of inequalities. A third criticism is that we do neither take into account a limited number of agents, nor the structure of connexions. Yet, many studies have shown that structure matters in the determination of employment when we consider information transfer through SN (see Calvò-Armengol et Jackson, 2004). This could also be the source of future researches. Finally, we work in a steady state equilibrium. Yet, inequalities may evolve. In studying dynamics, one could try explaining why the edge of the minority community in a labor pool matters as Patel and Vella (2007), Frijters & al (2005) and Munshi (2003) have shown.

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A.1. The choice of exogenous parameters

As Fontaine (2008), we choose the following value corresponding to the French economy:

If r is the daily interest rate, we choose $r = 0.00016$.

b, the daily job destruction rate is fixed such that $b = 0.0002$.

The bargaining power of workers x is as usual $x = 0.5$.

Productivity of workers is normalized to 1, then $y_1 = 1$.

The cost of a vacant job is estimated to 0.3, the $h = 0.3$.

We fix the global unemployment rate $u = 0.1$.

A.2. The choice of endogenous parameters without D

We know that $p_i = p(\theta)\left(1 + \delta \times \left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j} + \frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right)\right)$, we then have to find $p(\theta)$ et δ.

A.2.1. The probability $p(\theta)$

$f_i = \frac{p(\theta)}{\theta}\left(\frac{n_iu_i}{u+\delta(1-u)} + \delta\left(\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j} + \frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\right)\right)$ then (26)$\Longleftrightarrow$

$$\frac{p(\theta)}{\theta}\left(\frac{n_iu_i}{u+\delta(1-u)} + \delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j} \\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right) \times \left(y_1 - w_1^{(i)}\right) = \frac{r+b}{1-b}h$$

$\Longleftrightarrow$

$$\frac{\theta}{p(\theta)} = \frac{1-b}{(r+b)h}\left(y_1 - w_1^{(i)}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i + \delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j} \\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right)$$

We choose $M(u+\delta(1-u), v) = A(u+\delta(1-u))^{0.5}v^{0.5}$. Indeed, we fix u thanks to the parameter A which catch institutional unobserved characteristics of the matching technology. We then have $p(\theta) = \frac{M(u+\delta(1-u),v)}{u+\delta(1-u)} = \frac{A(u+\delta(1-u))^{0.5}v^{0.5}}{u+\delta(1-u)} = A\frac{v^{0.5}}{(u+\delta(1-u))^{0.5}} = \left(\frac{v}{u+\delta(1-u)}\right)^{0.5} = A\theta^{0.5}$, then $\frac{\theta}{p(\theta)} = \frac{\theta}{A\theta^{0.5}} = \frac{1}{A}\theta^{0.5} = \frac{1}{A^2}A\theta^{0.5} = \frac{1}{A^2}p(\theta)$. We then have

$$p(\theta) = A^2\frac{1-b}{(r+b)h}\left(y_1 - w_1^{(i)}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i + \delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j} \\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right)$$

A.2.2. The probability that an employee transmit a job offer δ

δ is fixed such that half of workers in the whole economy have found their job through network. The probability p is then specified such that

$$p = \underbrace{p(\theta)}_{\substack{\text{Formal}\\ \text{method}}} + \underbrace{\frac{u_i n_i}{u}\widetilde{\varphi}_i + \frac{u_j(1-n_i)}{u}\widetilde{\varphi}_j}_{\substack{\text{Informal}\\ \text{method}}}$$

so δ is chosen such that $p(\theta) = \frac{u_i n_i}{u}\widetilde{\varphi}_i + \frac{u_j(1-n_i)}{u}\widetilde{\varphi}_j$, then

$$\begin{aligned} p(\theta) \;=\; & \frac{u_i n_i}{u}\left(p(\theta)\times\delta\times\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j + 1 + (d_j-1)u_i}\right)\right) \\ & + \frac{u_j(1-n_i)}{u}\left(p(\theta)\times\delta\times\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i + 1 + (d_i-1)u_j}\right)\right) \end{aligned}$$

$\Longleftrightarrow$

$$p(\theta) = p(\theta)\times\delta\left(\begin{array}{c} \frac{u_i n_i}{u}\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j+1+(d_j-1)u_i}\right) \\ +\frac{u_j(1-n_i)}{u}\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i+1+(d_i-1)u_j}\right) \end{array}\right)$$

$\Longleftrightarrow$

$$\delta = \frac{1}{\frac{u_i n_i}{u}\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j+1+(d_j-1)u_i}\right) + \frac{u_j(1-n_i)}{u}\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i+1+(d_i-1)u_j}\right)} \tag{31}$$

A.2.3. The parameter A

$$u = \frac{b}{p(1-b)+b}$$

then

$$p = \frac{b(1-u)}{u(1-b)}$$

but we saw that

$$p = p(\theta) + \frac{u_i n_i}{u}\widetilde{\varphi}_i + \frac{u_j(1-n_i)}{u}\widetilde{\varphi}_j$$

then

$$p = p(\theta)\left(1 + \frac{u_i n_i}{u}\times\delta\times\left(\begin{array}{c}\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} \\ +\frac{d_i(1-u_j)}{u_j s_j+1+(d_j-1)u_i}\end{array}\right) + \frac{u_j(1-n_i)}{u}\times\delta\times\left(\begin{array}{c}\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} \\ +\frac{d_j(1-u_i)}{s_i u_i+1+(d_i-1)u_j}\end{array}\right)\right)$$

We then have

$$p = \frac{b(1-u)}{u(1-b)}$$

$\Longleftrightarrow$

$$p(\theta)\left(\begin{array}{c} 1+\frac{u_i n_i}{u}\times\delta\times\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j}+\frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right) \\ +\frac{u_j(1-n_i)}{u}\times\delta\times\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_ju_i}+\frac{d_j(1-u_i)}{s_iu_i+1+(d_i-1)u_j}\right)\end{array}\right)$$
$$=\quad\frac{b(1-u)}{u(1-b)}$$

but $p(\theta)=A^2\frac{1-b}{(r+b)h}\left(y_1-w_{1_i}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i+\delta\left(\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j}+\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\right)\right)$ then

$$A^2=\frac{b(1-u)}{\left(\begin{array}{c} u(1-b)\left(1+\delta\left(\begin{array}{c}\frac{u_in_i}{u}\times\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j}+\frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right)\\ +\frac{u_j(1-n_i)}{u}\times\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_ju_i}+\frac{d_j(1-u_i)}{s_iu_i+1+(d_i-1)u_j}\right)\end{array}\right)\right)\frac{1-b}{(r+b)h} \\ \times\left(y_1-w_{1_i}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i+\delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j}\\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right)\end{array}\right)}$$

If we take into account $\delta=\frac{1}{\frac{u_in_i}{u}\left(\begin{array}{c}\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j}\\ +\frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\end{array}\right)+\frac{u_j(1-n_i)}{u}\left(\begin{array}{c}\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_ju_i}\\ +\frac{d_j(1-u_i)}{s_iu_i+1+(d_i-1)u_j}\end{array}\right)}$

then we find

$$A^2=\frac{b(1-u)}{u(1-b)\,2\frac{1-b}{(r+b)h}\left(y_1-w_{1_i}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i+\delta\left(\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j}+\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\right)\right)}$$

A.3. The choice of endogenous parameters with D

A.3.1. The probability $p(\theta)$

We have $f_i=D\frac{p(\theta)}{\theta}\left(\frac{n_iu_i}{u+\delta(1-u)}+\delta\left(\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j}+\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\right)\right)$ then (26)$\Longleftrightarrow$

$$D\frac{p(\theta)}{\theta}\left(\frac{n_iu_i}{u+\delta(1-u)}+\delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j}\\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right)\times\left(y_1-w_1^{(i)}\right)=\frac{r+b}{1-b}h$$

$\Longleftrightarrow$

$$\frac{\theta}{p(\theta)}=\frac{D(1-b)}{(r+b)h}\left(y_1-w_1^{(i)}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i+\delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j}\\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right)$$

But we choose $M(u+\delta(1-u),v)=A(u+\delta(1-u))^{0.5}v^{0.5}$. Indeed we fix thanks to A that we defined before. We then have $p(\theta)=\frac{M(u+\delta(1-u),v)}{u+\delta(1-u)}=\frac{A(u+\delta(1-u))^{0.5}v^{0.5}}{u+\delta(1-u)}=$

$A\frac{v^{0.5}}{(u+\delta(1-u))^{0.5}} = \left(\frac{v}{u+\delta(1-u)}\right)^{0.5} = A\theta^{0.5}$, then $\frac{\theta}{p(\theta)} = \frac{\theta}{A\theta^{0.5}} = \frac{1}{A}\theta^{0.5} = \frac{1}{A^2}A\theta^{0.5} = \frac{1}{A^2}p(\theta)$. We find

$$p(\theta) = A^2\frac{D(1-b)}{(r+b)h}\left(y_1 - w_1^{(i)}\right)\frac{1}{u+\delta(1-u)}\left(n_iu_i + \delta\left(\begin{array}{c}\frac{n_i(1-u_i)}{u+\delta(1-u)}\frac{s_iu_i}{s_iu_i+d_iu_j} \\ +\frac{(1-n_i)(1-u_j)}{u+\delta(1-u)}\frac{d_ju_i}{s_ju_j+d_ju_i}\end{array}\right)\right)$$

A.3.2. A and δ when D exist

p, the average probability to find a job in the economy is defined as follow

$$p = \underbrace{\frac{u_in_i}{u}Dp(\theta) + \frac{u_j(1-n_i)}{u}p(\theta)}_{\substack{\text{Formal} \\ \text{method}}} + \underbrace{\frac{u_in_i}{u}D\widetilde{\varphi}_i + \frac{u_j(1-n_i)}{u}\widetilde{\varphi}_j}_{\substack{\text{Informal} \\ \text{method}}}$$

δ is then chosen such that $\frac{u_in_i}{u}Dp(\theta) + \frac{u_j(1-n_i)}{u}p(\theta) = \frac{u_in_i}{u}D\widetilde{\varphi}_i + \frac{u_j(1-n_i)}{u}\widetilde{\varphi}_j$, then

$$\begin{aligned} & \frac{u_in_i}{u}Dp(\theta) + \frac{u_j(1-n_i)}{u}p(\theta) \\ = & \frac{u_in_i}{u}D\left(p(\theta)\times\delta\times\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j} + \frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right)\right) \\ & +\frac{u_j(1-n_i)}{u}\left(p(\theta)\times\delta\times\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_ju_i} + \frac{d_j(1-u_i)}{s_iu_i+1+(d_i-1)u_j}\right)\right) \end{aligned}$$

$\Longleftrightarrow$

$$\begin{aligned} u_in_iD + u_j(1-n_i) = & \ u_in_iD\delta\times\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j} + \frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right) \\ & +u_j(1-n_i)\delta\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_ju_i} + \frac{d_j(1-u_i)}{s_iu_i+1+(d_i-1)u_j}\right) \end{aligned}$$

$\Longleftrightarrow$

$$\delta = \frac{u_in_iD + u_j(1-n_i)}{u_in_iD\times\left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_iu_j} + \frac{d_i(1-u_j)}{u_js_j+1+(d_j-1)u_i}\right) + u_j(1-n_i)\left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_ju_i} + \frac{d_j(1-u_i)}{s_iu_i+1+(d_i-1)u_j}\right)}$$

For A, we start from $p = \frac{b(1-u)}{u(1-b)}$, then

$$\begin{aligned}
& \frac{u_i n_i}{u} D p(\theta) + \frac{u_j(1-n_i)}{u} p(\theta) \\
& + \frac{u_i n_i}{u} D \left(p(\theta) \times \delta \times \left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i + d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j + 1 + (d_j-1)u_i} \right) \right) \\
& + \frac{u_j(1-n_i)}{u} \left(p(\theta) \times \delta \times \left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j + d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i + 1 + (d_i-1)u_j} \right) \right) \\
= & \frac{b(1-u)}{u(1-b)}
\end{aligned}$$

$\Longleftrightarrow$

$$\frac{p(\theta)}{u} \left(\begin{array}{c} u_i n_i D + u_j(1-n_i) + u_i n_i D \times \delta \times \left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j + 1 + (d_j-1)u_i} \right) \\ + u_j(1-n_i) \times \delta \times \left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i + 1 + (d_i-1)u_j} \right) \end{array} \right) = \frac{b(1-u)}{u(1-b)}$$

$\Longleftrightarrow$

$$p(\theta) \left(u_i n_i D + u_j(1-n_i) + \delta \left(\begin{array}{c} u_i n_i D \times \left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j + 1 + (d_j-1)u_i} \right) \\ + u_j(1-n_i) \times \left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i + 1 + (d_i-1)u_j} \right) \end{array} \right) \right) = \frac{b(1-u)}{(1-b)}$$

but $\delta = \frac{u_i n_i D + u_j(1-n_i)}{u_i n_i D \times \left(\frac{s_i(1-u_i)}{1+(s_i-1)u_i+d_i u_j} + \frac{d_i(1-u_j)}{u_j s_j+1+(d_j-1)u_i} \right) + u_j(1-n_i) \left(\frac{s_j(1-u_j)}{1+(s_j-1)u_j+d_j u_i} + \frac{d_j(1-u_i)}{s_i u_i+1+(d_i-1)u_j} \right)}$
then

$$p(\theta) = \frac{b(1-u)}{(1-b)\, 2 \left(u_i n_i D + u_j(1-n_i) \right)}$$

but $p(\theta) = A^2 \frac{1-b}{(r+b)h} (y_1 - w_{1_i}) \frac{1}{u+\delta(1-u)} \left(n_i u_i + \delta \left(\begin{array}{c} \frac{n_i(1-u_i)}{u+\delta(1-u)} \frac{s_i u_i}{s_i u_i + d_i u_j} \\ + \frac{(1-n_i)(1-u_j)}{u+\delta(1-u)} \frac{d_j u_i}{s_j u_j + d_j u_i} \end{array} \right) \right)$, we find

$$A^2 = \frac{b(1-u)}{(1-b)\, 2 \left(\begin{array}{c} u_i n_i D \\ + u_j(1-n_i) \end{array} \right) \frac{1-b}{(r+b)h} (y_1 - w_{1_i}) \frac{1}{u+\delta(1-u)} \left(n_i u_i + \delta \left(\begin{array}{c} \frac{n_i(1-u_i)}{u+\delta(1-u)} \frac{s_i u_i}{s_i u_i + d_i u_j} \\ + \frac{(1-n_i)(1-u_j)}{u+\delta(1-u)} \frac{d_j u_i}{s_j u_j + d_j u_i} \end{array} \right) \right)}$$